The Crazy World of Arthur L. Besse

A workshop on Einstein manifolds

October 4-6, 2023
at the University of Stuttgart

Program and Scientific talks

Organizers:
Brice Flamencourt (Universität Stuttgart)
Andrei Moroianu (Université Paris-Saclay)
Paul Schwahn (Universität Stuttgart/Université Paris-Saclay)
Uwe Semmelmann (Universität Stuttgart)

Program

A workshop on Einstein manifolds

Lectures: room 8.122

Welcome and coffee break: room 7.530

Wednesday, 04.10.2023

13:00 – 14:00	Welcome/ Opening
14:00 – 14:50	**Bernd Ammann** (Universität Regensburg) *Parallel spinors and a Hamilton-Jacobi equation*
15:00 – 15:50	**Klaus Kröncke** (KTH Royal Institute of Technology) *Scalar curvature rigidity, mass and stability of Einstein manifolds*
16:00 – 16:30	Coffee Break
16:30 – 17:00	**Albachiara Cogo** (Universität Tübingen) *Black Hole and Equipotential Photon Surface uniqueness in (n+1)-dimensional static vacuum spacetimes via Robinson's method*
17:15 - 17:45	**Carlos Shahbazi** (UNED Madrid) *The local moduli space of Einstein-Yang-Mills system*
18:15	Dinner (La Bruschetta - Campus Vaihingen)

Thursday, 05.10.2023

9:30 – 10:20	**Lorenz Schwachhöfer** (Technische Universität Dortmund) *Prescribing the curvature of torsion free connections*
10:30 – 11:00	Coffee Break
11:00 – 11:50	**Stuart James Hall** (Newcastle University) *Rigidity of SU_{2n+1} and some other symmetric spaces*
12:00 – 14:00	Lunch
14:00 – 14:30	**Diego Conti** (Universita di Pisa) *Indefinite Einstein solvmanifolds*
14:45 – 15:15	**Diego Artacho** (Imperial College London) *Invariant twisted spinors on homogeneous spaces*
15:30 – 16:00	**Mustafa Kalafat** (Universität Bonn) *Topology of Lie groups and Grassmannians with applications to Geometry*
16:00 – 16:30	Coffee Break and Photo
16:30 - 17:00	**Louis Yudowitz** (KTH Royal Institute of Technology) *Bubble Tree Convergence of Shrinking Ricci Solitons*
17:15 - 17:45	**Rodrigo Avalos** (Universität Potsdam) *Q-singular manifolds and the Einstein condition*
20:00	Conference Dinner (Alte Kanzlei Stuttgart City Centre)

Friday, 06.10.2023

9:30 – 10:20	**Lorenzo Foscolo** (University College London) *Instantons on ALF spaces and codimension-1 collapse*
10:30 – 11:00	Coffee Break
11:00 – 11:30	**Ivan Tulli** (University of Sheffield) *Instanton corrected quaternionic-Kähler metrics and their isometries*
11:45 – 12:35	**Christoph Böhm** (Universität Münster) *Non-compact Einstein manifolds with unimodular isometry group*
12:45	Lunch and Closing

Dinner information

For Wednesday evening, we have booked some tables at *La Bruschetta*, an Italian restaurant located on the university campus. To get there from the math building, cross two brigdes and walk a few steps. The reservation is from 18:15 on but you may of course join at a later time.

URL: ristorantebruschetta.de
Address: Pfaffenwaldring 62, 70569 Stuttgart

The proper conference dinner will take place on Thursday, 20:00, at *Alte Kanzlei*, a traditional Swabian restaurant located in the city center. From the university, we recommend taking the S-Bahn to either Stadtmitte or Hauptbahnhof.

URL: alte-kanzlei-stuttgart.de
Address: Schillerplatz 5, 70173 Stuttgart

Abstracts

Bernd Ammann (Universität Regensburg)
Parallel spinors and a Hamilton-Jacobi equation

Work in collaboration with Klaus Kröncke, related to older work with Olaf Müller, Jonathan Glöckle, Hartmut Weiß und Frederik Witt.
During the last years, a tight relationship between Lorentzian manifolds with a parallel spinor and families of Riemannian manifolds with parallel spinors emerged. Inspiring work by Baum, Leistner and Lischewski showed that if N^{n+1} is a Lorentzian manifold of dimension $n+1$ with a parallel lightlike spinor, and if M^n is a spacelike hypersurface, then M carries a foliation by hypersurfaces Q_s such that the induced Riemannian metrics g_s on Q_s carry a parallel spinor. The metric on M then locally can be written as $u^{-2}ds^2+g_s$, $u \in C^\infty(M;\mathbb{R}_+)$. Let us restrict to the case that Q_s is closed.
In collaboration with Klaus Kröncke and Olaf Müller, we constructed a converse to this. Starting with a path g_s of Riemannian metrics on a closed spin manifold Q, with a parallel spinor on (Q, g_s), we constructed a Lorentzian N^{n+1} as above, the Riemannian metric on a hypersurface M^n is then of the form ds^2+g_s, which is apparently more special than the decomposition given above. It is thus natural to ask whether one cleverly choose the spacelike hypersurface M in the Baum–Leistner–Lischewki construction such that $ds^2 + g_s$ is obtained. We answer this question affirmatively, locally in the parameter s. The problem is reduced to a Hamilton–Jacobi equation that we will then solve locally in s.

Diago Artacho (Imperial College London)
Invariant twisted spinors on homogeneous spaces

It is well-known that the existence of a non-zero Killing spinor on a spin manifold implies that the manifold is Einstein. This is an example of how special spinors encode geometric

properties of manifolds. However, not every oriented Riemannian manifold is spin (e.g., complex projective spaces of positive even complex dimension). The idea of the talk is to introduce spin^r structures, which include the usual spin, spin^c and spin^h structures for $r = 1$, 2 and 3 respectively. These structures allow us to talk about twisted spinors on every oriented Riemannian manifold, even if it is not spin, spin^c or spin^h. The focus will be on the form these structures take on homogeneous spaces, and some examples will be given (spheres and projective spaces). This is joint work with Jordan Hofmann and Marie-Amélie Lawn.

Rodrigo Avalos (Universität Potsdam)

Q-singular manifolds and the Einstein condition

In this talk we intend to present recent results related to rigidity of asymptotically Euclidean (AE) Q-singular spaces. In particular, these spaces contain Riemannian manifolds which satisfy a fourth-order analogue of the Einstein condition, satisfied by Einstein manifolds, and which we refer to as J-Einstein manifolds. A priori, the analysis of J-Einstein manifolds is analytically more challenging, involving fourth order geometric partial differential equations, but several interesting properties of Einstein manifolds are still retained by this wider family of Einstein-type manifolds. In this talk we shall focus on AE manifolds, showing that this fourth order J-tensor retains optimal controls on the decay of the metric tensor at infinity, and also that J-flat Yamabe positive AE manifolds exhibit the same rigidity properties as Ricci-flat AE manifolds do.

Christoph Böhm (Universität Münster)

Non-compact Einstein manifolds with unimodular isometry group

We show that a negative Einstein manifold admitting a proper isometric action of a connected unimodular Lie group with compact, possibly singular, orbit space splits isometrically as a product of a symmetric space and a compact negative Einstein manifold. The proof involves the theory of polar actions, Lie-theoretic arguments and maximum principles. This is joint work with Ramiro Lafuente.

Carla Cederbaum/Albachiara Cogo (Universität Tübingen)

Black Hole and Equipotential Photon Surface uniqueness in n+1-dimensional static vacuum spacetimes via Robinson's method

In this joint work with Leandro and Dos Santos, we generalize to any dimension $n + 1$ Robinson's divergence formula used to prove the uniqueness of (3+1)-dimensional static black holes. To this end, we use a tensor first introduced by Cao and Chen for the analysis and classification of Ricci solitons. We thereby prove the uniqueness of black holes and of equipotential photon surfaces in the class of asymptotically flat $(n + 1)$-dimensional static vacuum space-times, provided the total scalar curvature of the horizon is properly bounded from above. In the black hole case, our results recover those of Agostiniani and Mazzieri and partially re-establish the results by Gibbons, Ida, and Shiromizu, and Hwang and finally by Raulot in the case of a spin manifold; in the photon surface case,

the results by Cederbaum and Galloway can also be proven. Our proof is not based on the positive mass theorem and avoids the spin assumption.

Diego Conti (Università di Pisa)

Indefinite Einstein solvmanifolds

In consequence of the proof of the Aleksveesky conjecture by Böhm and Lafuente, we know that homogeneous Riemannian Einstein manifolds of negative curvature are solvmanifolds. Much is known regarding the structure of Riemannian Einstein solvmanifolds, thanks to earlier work of Heber, Lauret, Nikolayevsky and others. The indefinite case is much more flexible. I will illustrate some peculiar phenomena which only occur for Einstein solvmanifolds of indefinite signature; the solvmanifold can be a nilpotent Lie group, the Ricci-flat case is nontrivial, the standard condition may fail, the Iwasawa condition may fail, and the Ricci operator of an underlying nilsoliton may be nondiagonalizable. I will also discuss some constructive aspects. Finally, I will discuss special geometries associated with these Einstein metrics, in particular those defined by a Killing spinor. This is joint work with Federico A. Rossi and Romeo Segnan Dalmasso.

Lorenzo Foscolo (University College London)

Instantons on ALF spaces and codimension-1 collapse

I will discuss joint work with C. Ross aimed at describing the behaviour of Yang-Mills instantons on ALF spaces in the collapsed limit where these 4-dimensional geometries are close to a 3-dimensional collapsed limit space. In this limit, the instantons are well approximated by superpositions of simple localised building blocks constructed out of monopoles on $\mathbb{R}^3$ (with singularities). I will then discuss how the construction can be used to describe the asymptotics of the hyperkähler L^2-metric on the moduli space of instantons, providing examples of a class of higher dimensional non-compact hyperkähler spaces with an asymptotic geometry, sometimes referred to as QALF, that generalises to higher dimensions the geometry of 4-dimensional ALF spaces.

Stuart James Hall (Newcastle University)

Rigidity of SU_{2n+1} and some other symmetric spaces

I will report on joint work with Wafaa Batat, Tommy Murphy and James Waldron where we show that certain infinitesimal Einstein deformations of SU_{2n+1} are not integrable to second order. If I have time, I'll talk about such deformations on the complex Grassmannians, some of which were recently shown to be non-integrable by Nagy and Semmelmann.

Mustafa Kalafat (Universität Bonn)

Topology of Lie groups and Grassmannians with applications to Geometry

Riemannian manifolds with G_2 holonomy is a special class of Einstein manifolds. In this talk, we give a survey of various results about the topology of oriented Grassmannian bundles related to the exceptional Lie group G_2. Some of these results are new. One often encounters these spaces when studying submanifolds of manifolds with calibrated geometries. As an application, we deduce the existence of certain special 3- and 4-dimensional submanifolds of G_2 holonomy Riemannian manifolds with special properties. These are called Harvey–Lawson (HL) pairs. Another application is to the free embeddings. We show that if there is a coassociative-free embedding of a 4-manifold into the Euclidean 7-space then the signature vanishes along with the Euler characteristic.

Klaus Kröncke (KTH Royal Institute of Technology)

Scalar curvature rigidity, mass and stability of Einstein manifolds

An Einstein manifold is called scalar curvature rigid if there are no compactly supported volume-preserving deformations of the metric which increase the scalar curvature. We give various characterizations of scalar curvature rigidity for closed as well as for open Einstein manifolds which are related to spectral properties of the Lichnerowicz Laplacian. For asymptotically hyperbolic manifolds, we construct a new mass invariant, called the volume-renormalized mass, which we use to characterize scalar curvature rigidity and stability of Poincaré-Einstein manifolds. This talk is based on joint work with Mattias Dahl and Stephen McCormick.

Lorenz Schwachhöfer (Technische Universität Dortmund)

Prescribing the curvature of torsion free connections

This is on joint work with Efrain Basurto. We consider the question of how freely the curvature of a torsion free connection may be prescribed locally. For this, we use a power series ansatz which leads to an alternative interpretation of the involutivity of the underlying exterior differential system that has been described by R. Bryant. With this, we can give a different interpretation of the local moduli space of these connections.

Carlos Shahbazi (Universidad UNED Madrid)

The local moduli space of Einstein-Yang-Mills system

I will describe the construction of the local Kuranishi model for the moduli space of solutions (g, A) of the coupled Einstein-Yang-Mills equations on a principal bundle P with compact simple structure group G over a compact manifold M, where g is a Riemannian metric on M and A is a connection on P. Time permitting, I will discuss some applications to the rigidity of Einstein–Yang–Mills solutions and the coupled deformation problem of stable holomorphic bundles over K3 surfaces.

Ivan Tulli (University of Sheffield)

Instanton corrected quaternionic-Kähler metrics and their isometries

Einstein metrics appear in several different contexts in physics and math. Of particular interest for this talk are the Einstein metrics coming from the supergravity (SUGRA) c-map construction and its instanton corrected versions. The SUGRA c-map associates to a projective special Kähler manifold a 1-parameter family of non-compact quaternionic-Kähler (QK) manifolds. Furthermore, in the string theory literature certain instanton corrections of the c-map metric have been studied, all of which preserve the QK property. Our aim in this talk is to present a mathematical treatment of such instanton corrected c-map metrics. More precisely, by enriching the data of the special Kähler manifold with a certain compatible variation of BPS structures, we construct an "instanton corrected" c-map metric. We also discuss certain isometries of such metrics, including the highly non-trivial $\mathrm{SL}(2,\mathbb{Z})$ "S-duality" isometry. This is joint work with V. Cortés (arXiv:2306.01463, arXiv:2202.03121 and arXiv:2105.09011), based on previous work in the physics literature by S. Alexandrov, S. Banerjee, B. Pioline and F. Saueressig (and many others).

Louis Yudowitz (KTH Royal Institute of Technology)

Bubble Tree Convergence of Shrinking Ricci Solitons

Introduced by Richard Hamilton in 1982, Ricci flow has been used to solve a variety of problems in geometry and topology. A vital part of such proofs is a good understanding of finite time singularities. While we have such an understanding in dimensions 2 and 3, singularity models in higher dimensions are still relatively mysterious. This is partially due to the existence of singularity models which are singular themselves. In this talk, we will prove bubble tree convergence of certain shrinking singularity models, which involves a detailed analysis of the singular set when it consists of isolated points. As a consequence, we will recover any topology lost due to the formation of the singular points, as well as prove a qualitative classification result. The above results also apply to positive Einstein manifolds, as they are special cases of Ricci shrinkers. This is all based on a joint work with Reto Buzano.

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Bernd Ammann (Regensburg)
Christoph Böhm (Münster)
Lorenzo Foscolo (UCL)
Stuart James Hall (Newcastle)
Klaus Kröncke (KTH, Stockholm)
Lorenz Schwachhöfer (Dortmund)

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