Oberseminar Geometrie und Topologie

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The large scale geometry and degenerations of Higgs bundle moduli spaces

21. November 2017 - 16 Uhr
Seminarraum IGT, Raum 7.530, Pfaffenwaldring 57

Abstract: In the first part of this talk I will explain the results from recent joint work with Rafe Mazzeo, Hartmut Weiß and Frederik Witt on the asymptotics of the natural L^2-metric G_{L^2} on the moduli space $\mathcal{M}$ of rank-2 Higgs bundles over a Riemann surface X as given by the set of solutions to Hitchin's self-duality equations

$$\begin{cases} 0 = \bar{\partial}_A \Phi \\ 0 = F_A + [\Phi \wedge \Phi^*] \end{cases}$$

for a unitary connection A and a Higgs field Φ on X. I will show that on the regular part of the Hitchin fibration $(A, \Phi) \mapsto \det \Phi$ this metric is well-approximated by the semiflat metric G_{sf} coming from the completely integrable system on $\mathcal{M}$. This result confirms some aspects of a more general conjectural picture due to Gaiotto, Moore and Neitzke. Its proof is based on a detailed understanding of the ends structure of $\mathcal{M}$. The analytic methods used there in addition yield a complete asymptotic expansion of the difference $G_{L^2} - G_{\mathrm{sf}}$ in terms of some "radial" variable t, with leading order term decaying at polynomial rate as $t \to \infty$.

The second part of the talk focusses on Higgs bundles on families of Riemann surfaces degenerating into a surface X_0 with nodes. There I will present a desingularization result for solutions of the self-duality equations on X_0, which builds on spectral estimates for Dirac-type operators on manifolds with cylindrical ends.

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