Institut für Geometrie und Topologie

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Some aspects about the Riemannian Curvature and its covariant derivatives

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Raum 7.530

Abstract:
(i) Let X be a vector field whose curl is zero everywhere in a simply connected set. Then X is a gradient of a function f. As a system of partial differential equations, the problem is whether the following has a solution

$$\frac{\partial f}{\partial x_i} = X_i, i = 1,2,3.$$

The isometric embedding problem can also be written as a system of partial differential equations

$$\sum_{k=1}^{4} \frac{\partial f_i}{\partial x^i} \frac{\partial f_i}{\partial x^j} = g_{i,j}, i,j = 1,2,3,$$

where f is the immersion and $g_{i,j}$is the coefficients of the metric tensor.
Is there a "curl"=0 for this problem too?

(ii) A locally symmetric space are naturally a semi-symmetric space. It naturally follows from $\nabla_X R = 0$ that $R_{X,Y} R = 0$. Conversely, there are examples where $R_{X,Y} R = 0$ but $\nabla_X R \neq 0$ [Nomizu 1967].
We can write $R_{X,Y} R = \nabla^2_{X,Y} R - \nabla^2_{Y,X} R$ and then also
$\nabla^2_{X,Y} R = \frac{1}{2} R_{X,Y} R + \frac{1}{2} S^2_{X,Y} R$ where $S^2_{X,Y} R = \nabla^2_{X,Y} R + \nabla^2_{Y,X} R$.
This decomposition of $\nabla^2_{X,Y} R$ is mathematically correct but it has a fundamental problem.

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