

Amann, Manuel (Universität Augsburg)

Fibrations with torus fibres and the toral rank conjecture

The toral rank conjecture speculates that the sum of the Betti numbers of a closed manifold admitting a free action of a torus of rank r is bounded from below by 2^r . Clearly, such an action yields a torus bundle, and, more generally, the same cohomological bound is conjectured for total spaces of suitable topological torus fibrations by Félix–Oprea–Tanré.

In this talk we illustrate that this generalised toral rank conjecture cannot hold by providing various different counter-examples to it (for each rank $r \geq 5$). In particular, we show that there are sequences of smooth nilpotent fibre bundles of nilmanifolds with fibre a torus of rank r such that the quotient of the total dimensions of the cohomologies of total space and fibre even converges to 0 with r tending to infinity.

Similar to recent examples by Walker (of a completely different nature) this shows that the toral rank conjecture is not likely to follow from “weaker conjectures or structures”.

Díaz-Ramos, Carlos (Universidade de Santiago de Compostela)

Homogeneous Lagrangian foliations in complex spaces forms

In this talk I will present the classification of holomorphic isometric actions on complex space forms all whose orbits are Lagrangian submanifolds. I will show that the only examples are Lagrangian affine subspace foliations of complex Euclidean spaces, and Lagrangian horocycle foliations of complex hyperbolic spaces.

This is a joint work with M. Domínguez-Vázquez and T. Hashinaga

Domínguez Vázquez, Miguel (Universidade de Santiago de Compostela)

Extension of submanifolds in symmetric spaces of noncompact type

In this talk I will present a simple method to extend submanifolds from certain Euclidean spaces embedded in a symmetric space of noncompact type to the whole ambient symmetric space, in such a way that the codimension, the mean curvature, and other geometric properties are preserved. As a direct application, we will construct the first examples of inhomogeneous isoparametric hypersurfaces in every symmetric space of noncompact type and rank higher than two. Surprisingly, these examples have nonempty focal set. This is based on a joint work with Víctor Sanmartín-López.

Gonzalez Alvaro, David (Universidad Politécnica de Madrid)

Manifolds of $\text{Ric}_k > 0$

The curvature conditions $\text{Ric}_k > 0$ on n -dimensional manifolds interpolate between positive sectional curvature (when $k = 1$) and positive Ricci curvature (when $k = n - 1$). In this talk we will review their definitions and context, and explain how to construct manifolds of $\text{Ric}_k > 0$ with k as small as possible. This is based on joint work with Miguel Domínguez-Vázquez and Lawrence Mouillé.

Kalafat, Mustafa (University of Bonn)

On special submanifolds of the Page space

Page manifold is the underlying differentiable manifold of the complex surface, obtained out of the process of blowing up the complex projective plane, only once. This space is decorated with a natural Einstein metric, first studied by D. Page in 1978. It is a cohomogeneity-1 manifold. In this talk, we study some classes of submanifolds of codimension one and two in the Page space. These submanifolds are totally geodesic. We also compute their curvature and show that some of them are constant curvature spaces. Finally, we give information on how the Page space is related to some other metrics on the same underlying smooth manifold. Related paper may be accessed from the following,

Kalafat, Mustafa; Sarı, Ramazan. On special submanifolds of the Page space. *Differential Geom. Appl.* 74 (2021), 101708, 13 pp.

Lorenzo-Naveiro, Juan Manuel (Universidade de Santiago de Compostela)

Polar actions on some noncompact symmetric spaces

An isometric action on a Riemannian manifold is said to be polar if it admits sections, that is, submanifolds that meet every orbit perpendicularly. Such actions provide a generalization to common situations arising in geometry, such as the system of polar coordinates on the plane or the spectral theorem for symmetric matrices. As of today, it is an open problem to classify polar actions on several families of ambient manifolds.

During this talk, we will focus on the classification problem on symmetric spaces of noncompact type. The deep connection between the theory of symmetric spaces and real semisimple Lie groups allows us to reformulate the geometric properties of polar actions in almost purely algebraic terms. With this relationship in mind, we will discuss the problem of determining all polar actions on $\mathrm{SL}(3, \mathbb{R})/\mathrm{SO}(3)$, the space of volume preserving positive definite linear maps over $\mathbb{R}^3$.

Martín-Merchan, Lucía (Università di Torino)

A compact non-formal closed G_2 manifold with $b_1 = 1$

A G_2 structure on a 7-dimensional Riemannian manifold determined by a certain type of 3-form φ . These are classified into 16 types according to PDEs involving φ ; for instance, the G_2 structure is torsion-free if φ is parallel, closed if φ is closed and cocalibrated if φ is co-closed. This talk contributes to understanding topological properties of compact manifolds with a closed G_2 structure that cannot be endowed with any torsion-free G_2 structure. Namely, we construct such a manifold that is non-formal and has first Betti number $b_1 = 1$. The starting point is a nilmanifold (M, φ) with a closed G_2 structure that admits an involution preserving φ such that the quotient $M/\mathbb{Z}_2$ is a non-formal orbifold with $b_1 = 1$. Then we perform a resolution of these singularities obtaining a manifold endowed with a closed G_2 structure; we finally prove that the resolution verifies the same topological properties and do not admit any torsion-free G_2 structure.

Otero-Casal, Tomás (Universidade de Santiago de Compostela)

Homogeneous hypersurfaces of symmetric spaces of noncompact type

A hypersurface of a Riemannian manifold is called homogeneous if it is an orbit of an isometric action on the ambient manifold. Thus, the classification of homogeneous hypersurfaces on a given Riemannian manifold amounts to the study of which subgroups of isometries act on this manifold with cohomogeneity one.

We continue the study of cohomogeneity one actions on symmetric spaces of noncompact type, generalizing the procedure proposed by Berndt and Tamaru for classifying such actions on irreducible symmetric spaces of noncompact type. It turns out that a cohomogeneity one action on a (not necessarily irreducible) symmetric space of noncompact type can always be constructed (up to orbit equivalence) in one of three distinct ways: By considering codimension one subgroups of the solvable model of the symmetric space, by extending cohomogeneity one actions on certain totally geodesic submanifolds, or by a process known as nilpotent construction.

In this way, we are able to give the first classification results of cohomogeneity one actions on some symmetric spaces of noncompact type and rank greater than two.

This is a joint work with José Carlos Díaz-Ramos and Miguel Domínguez-Vázquez.

Rodríguez-Vázquez, Alberto (Universidade de Santiago de Compostela)

Totally geodesic submanifolds in products of rank one symmetric spaces

Totally geodesic submanifolds in symmetric spaces are those submanifolds with the simplest geometry and they admit a nice algebraic characterization in terms of Lie triple systems.

In this talk, I will report on an ongoing work where a classification of totally geodesic submanifolds in products of symmetric spaces of rank one is given.

Sanmartín-López, Victor (Universidad Politécnica de Madrid)

Codimension one Ricci solitons in Iwasawa groups

A symmetric space of non-compact type is isometric to a solvable Lie group AN endowed with a left-invariant metric. This solvable Lie group AN is constructed as a semidirect product of an abelian Lie group A and a nilpotent Lie group N . Investigating simply connected Ricci soliton solvmanifolds (in particular, Einstein solvmanifolds) is closely related to studying Lie subgroups of N whose induced metric is a Ricci soliton.

In this talk, we will address the investigation and state the classification result of codimension one Lie subgroups of N whose induced metric is a Ricci soliton.

Siffert, Anna (Universität Münster)

Stability of harmonic self-maps between cohomogeneity one manifolds

In a recent paper, Püttmann and Siffert studied systematically harmonic self-maps of cohomogeneity one manifolds. In this talk we investigate the corresponding Jacobi equation describing the stability of such harmonic self-maps. If time permits we will discuss a related problem for minimal surfaces. This is joint work with Volker Branding.

Solonenko, Ivan (King's College London)

Homogeneous hypersurfaces in the noncompact complex two-plane Grassmannians

I will talk about the classification of cohomogeneity-one actions on and thus homogeneous hypersurfaces in the symmetric spaces of noncompact type $M = \text{Gr}^*(2, \mathbb{C}^{n+4})$, $n > 0$. Since the classification of C1-actions without singular orbits or with a totally geodesic singular orbit has been known for a while from the works of Berndt and Tamaru, I will focus primarily on actions with a non-totally-geodesic singular orbit. Specifically, I will explain why all such actions arise from C1-actions on the totally geodesic submanifolds $\mathbb{R}H^3$ and $\mathbb{C}H^{n+1}$ of M (a.k.a. boundary components) via the so-called canonical extension procedure. Since M is both a Hermitian symmetric space and a quaternion-Kähler manifold, the interplay between these structures will play an important role in the classification.

Zarei, Masoumeh (Universität Münster)

Intermediate Ricci, homotopy, and submanifolds of symmetric spaces

In the spirit of combining Riemannian geometry, topology, and algebra when studying symmetric spaces, we introduce a new approach to the study and identification of submanifolds of simply-connected symmetric spaces of compact type based upon the computation of their *k-positive Ricci curvature*. We then apply the “generalized connectedness lemma” by Guijarro-Wilhelm to certain classes of submanifolds of symmetric spaces, including totally geodesic ones, to show that within certain codimension ranges such submanifolds have the same “Cartan type” as their ambient spaces, (possibly) up to product with spheres. This is joint work with Manuel Amann and Peter Quast.